Properties of Logarithms

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- **LOG of 1 is 0**: Given the logarithmic function f(x) = $LOG_a x$, f(1) = 0.In other words, **$LOG_a 1 = 0$ for any legitimate exponential base a**.

- **LOG_a of a is 1**: Given the logarithmic function f(x) = $LOG_a x$, f(a) = 1.In other words, **$LOG_a a = 1$ for any legitimate exponential base a**.

- **Product Rule for Logs**: Given the logarithmic function f(x) = $LOG_a x$, f(UV) = f(U) + f(V).In other words, **$LOG_a(UV) = LOG_a U + LOG_a V$** for any legitimate exponential base a.

- **Quotient Rule for Logs**: Given the logarithmic function f(x) = $LOG_a x$, f(U/V) = f(U) - f(V).In other words,

  $$\mathrm{Log}_a\left(\frac{U}{V}\right) = \mathrm{Log}_a U - \mathrm{Log}_a V$$

  for any legitimate exponential base a.

- **Power Rule for Logs**: Given the logarithmic function f(x) = $LOG_a x$, f(x^N) = N●f(x).In other words, **$LOG_a(x^N) = N \bullet LOG_a x$**.

- **Change of Base Rule for Logs**: Given the logarithmic function f(x) = $LOG_a x$, it is true, for any legitimate bases a and b, that

  $$\mathrm{Log}_a x = \frac{\mathrm{Log}_b x}{\mathrm{Log}_b a}$$

- **Inverse Property**: Given the exponential function f(x) = a^x, the inverse of f(x) is the logarithmic function form is $f^{-1}(x) = LOG_a x$.

  Also, since $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$, we can say that

  $$a^{\log_a x} = x \quad \text{and}$$

  $$\log_a(a^x) = x$$

Example: Use properties of logs to rewrite 3LN(x) – (1/2)LN(2) + LN(z) by combining log terms into a single log.

3LN(x) – (1/2)LN(2) + LN(z)	
= LN(x^3) – LN($2^{1/2}$) + LN(z)	Applying the **Power Rule For Logs** twice
= LN(x^3) – LN(√2) + LN(z)	Applying the rule for rational exponents
= LN**[**(x^3)/(√2)**]** + LN(z)	Applying the **Quotient Rule For Logs**
= LN **[**(x^3)/(√2)**]**•z**]**	Applying the **Power Rule For Logs**
= LN **[**(z•x^3)/(√2)**]**	Multiply the fraction by z

Example: Use properties of logs to rewrite the expression shown below as a sum or difference of logs or multiples of logs.

$$LN\left(\frac{2x^3}{\sqrt{x}}\right)$$

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= LN $(2x^3)$ – LN($\sqrt{x}$)	Application of **Quotient Rule For Logs**
= LN $(2x^3)$ – LN($x^{1/2}$)	Definition of rational exponents
= LN(2) + LN(x3) – LN($x^{1/2}$)	Application of **Product Rule For Logs**
= LN(2) + 3LN(x) – ½ LN(x)	Application of **Power Rule For Logs**

Example: Rewrite $3^X = 28$ in log form. Then rewrite this log as a ratio of two base-10 logs and also as two base-e logs. Then evaluate your ratios using a scientific calculator in order to solve for x.

The equivalent log form of $3^X = 28$ is $LOG_3(28) = x$.

Now, applying the Change of Base Formula, we get

$$LOG_3(28) = \frac{LOG_{10}(28)}{LOG_{10}(3)}$$

and also $$LOG_3(28) = \frac{LN(28)}{LN(3)}$$

Using a scientific calculator, for either expression results in 3.033103256…

So, we conclude that x = 3.033, rounded to the thousandth, which is what we would expect, since $3^3 = 27$, and we are finding the power of 3 resulting in 28.