Introduction To Functions

A function from the set of x-values to the set of y-values is a rule or formula where x-values are input and the function assigns exactly 1 y-value to each x-value that is input.

The set of all x-values that are input is called the DOMAIN.

The set of all y-values that correspond to the given x-values is called the RANGE.

Examples of equations that represent y as a function of x, and some that don't:

$y = 2x + 1$ is a function of x since each x-value input results in only 1 y-value.

$|y| = x$ is NOT a function of x since x=9 corresponds to both y = 9 *and* y = -9.

$y = x^2$ is a function of x since each x-value input results in only 1 y-value.

$y^2 = x$ is NOT a function of x since x = 4 results in y = 2 *and* y = -2.

$y = \pm\sqrt{x}$ is NOT a function of x since x = 4 results in y = 2 *and* y = -2.

Function Notation
We may write a formula that defines a function with what is called "function notation". We replace y with f(x).

We call f(x) "f of x" and it means "the y=value when x is input."

Example: If $f(x) = 2x + 1$, what is $f(3)$, $f(-5)$, and $f(A)$?

$f(3) = 2(3) + 1 = 7$
$f(-5) = 2(-5) + 1 = -9$
$f(A) = 2(A) + 1$ or $2A + 1$

To use Function Notation, just remember to replace each x in the original formula with whatever is after f within the parentheses!

Example: Given $f(x) = x^2 + 1$, evaluate and simplify the expression below.

$$\frac{f(x+h) - f(x)}{h}$$

Here, you are probably wondering, "What is h?" We don't know what h is, so we simply carry it through all the operations as a variable.

Our first step in this problem is to evaluate the two expressions in the numerator. Then plug in these results and simplify. CONTINUED . . .

$\frac{f(x+h) - f(x)}{h}$	Given
$f(x + h) = (x + h)^2 + 1$ $= x^2 + xh + xh + h^2 + 1$ $= x^2 + 2xh + h^2 + 1$	Use the definition of function notation. Use the Distributive Property, and combination of like terms to simplify.
$f(x) = x^2 + 1$ which is the given formula	Use the definition of function notation.

Now, substitute in the above results to get

$$\frac{x^2 + 2xh + h^2 + 1 - (x^2 + 1)}{h}$$

Apply the Distributive Property to add –1 times the second quantity, add like terms, and simplify to get

$$\frac{2xh + h^2}{h}$$

Apply the Distributive Property to factor, and then Cancel Factors of h to get

$$\frac{h(2x + h)}{h}$$

$= 2x + h$

The previous example illustrates a use of function notation used in Calculus. The quotient shown represents the derivative of f(x) when the value "h" is allowed to approach zero.

Notations Other Than f(x)

Besides f(x), an variable may precede (x) in the notation. And in fact, variables other than x may, and are, often used. Here are some examples:

Examples:

g(x), h(x), and P(x) are often used instead of or in addition to f(x).
P(t) is often used to give the population P, as a function of time t.

So if $P(t) = 3t^2$ represented the population P as a function of t in years, then P(5) would equal $3 \bullet 5^2 = 75$, which would indicate that the population is 75 at t=5 years.

Two-Part Functions
Some functions are defined in two or more parts. For example, the function

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \geq 1 \\ x + 1 & \text{if } x < 1 \end{cases}$$

uses the formula $y = x^2+1$ for x values ≥ 1 and uses the formula y = x+1 for x values < 1. So for example $f(4) = 4^2 + 1 = 16$ and f(-1) = -1 + 1 = 0.

Finding The Domain of a Function
The domain of a function consists of the largest set of input-values that results in a real output value of the function, unless otherwise restricted. With a function where y is a function of x, we are looking for the largest set of x-values that makes y a real number.

Example: Find the domain of f(x) = 1 / (x – 1).

In this case, we are finding the largest set of x-values that makes the y-value defined as y = 1/(x-1) a real number.

Since x=1 results in division by zero, we know that x=1 is not part of the domain. All other real numbers result in a real y-value. Therefore **the domain of f(x) is All Real Numbers Except x = 1**.

Here are equivalent ways to state this domain **All Real Numbers Except x = 1** :

- (- ∞, 1) U (1, ∞)
- All Reals, x ≠ 1
- x ≠ 1

HINT: The way to find the domain of a function is to look for values of x that don't result in a real value of y. The domain is usually All Real Numbers Except For These Values. For example, in y= √x, all negative values of x will result in a non-real y-value. So the domain will be: All Reals *Except* Negative Numbers *or equivalently* All x ≥ 0.

Restricting a Domain For Certain Applications
In many applications, the input values for our function are restricted. For example, if we have a formula (function) that calculates the weight of a fish using it's length as an input value, the domain will not include negative values since a fish can not have a negative length. Or, if our input value consists of number of people visiting a shopping mall, we would restrict the domain to positive counting numbers since we can have neither negative numbers of people nor fractional numbers of people.