Function Graphing Rules

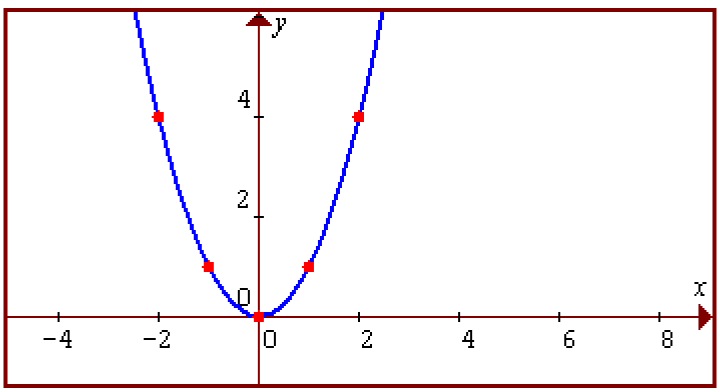
Are you tired of calculating points? The rules in this section will allow you to graph something like $y = 2(x - 4)^2 - 3$ without calculating a single point! Also, you will learn rules that will give you insight into the symmetry of graphs like $y = x^4 - 2x^2$ without calculating a point!

Even Functions

A function is defined as **even** if opposite real values of x result in the same y-value. In other words, a function is even if f(a) = f(-a) for any real value of "a". For example, $f(x) = x^2$ is an even function since $f(a) = a^2$ and $f(-a) = a^2$.

Graphs of Even Functions

Since opposite values of x result in the same y-value, the **graph of an even function will always have symmetry with respect to the y-axis**. For example, the graph of $y=x^2$ is shown below.



Example: Plot the graph of $y = x^4 - x^2$. Then determine whether or not this is an even function and note any symmetry of the graph.

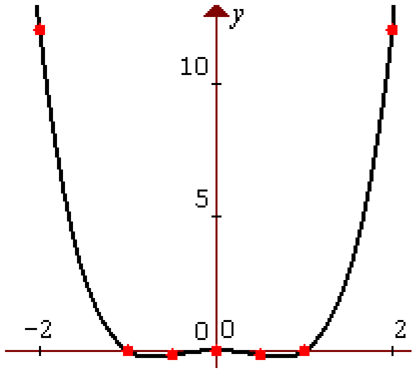
Letting x=0 results in y=0, so our y-intercept is (0,0).
We can find the x-intercepts by letting y=0 and solving $0 = x^4 - x^2$.

Factor $0 = x^4 - x^2$ with the Distributive Property to get
$0 = x^2(x^2 - 1) = x^2(x + 1)(x - 1)$.

Apply the Zero Product Law to get solutions
$x^2 = 0$, $x + 1 = 0$, and $x - 1 = 0$.

Extract square roots and apply the Addition Property of Equality to solve and get
x = 0, x = -1, x = 1.

This results in x-intercepts of (0,0), (-1,0), and (1,0). Plot these points and find more points on each side of each x-intercept. Draw a smooth curve through as shown on the following page.



x	y
1	0
-1	0
2	12
-2	12
1/2	-3/16
-1/2	-3/16
0	0

It becomes clear that this is an even function as x=2 and x=-2 both resulted in y=12 and both x=1/2 and x=-1/2 both resulted in y = -3/16. Also, **both terms of $y = x^4 – x^2$ involve even powers of x, making opposite input values of x result in the same y-value**.

The graph of this function clearly exhibits symmetry with respect to the y-axis.

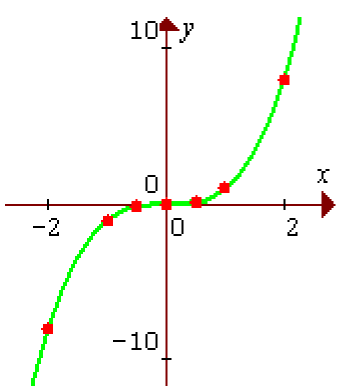
TIP: Use the knowledge of even functions to give you a mental picture of the graph before you even plot it. By noting that $y = x^4 – x^2$ is even due to its even powers of x, we would know that its graph will be a mirror image of itself about the y-axis.

Odd Functions

A function is defined as **odd** if opposite real values of x result in opposite y-values. In other words, a function is odd if f(a) = -f(-a) for any real value of "a". For example, $f(x) = x^3$ is an odd function since $f(a) = a^3$ and $f(-a) = (-a)^3 = -a^3$.

Graphs of Odd Functions

Since opposite values of x result in the opposite y-value, the **graph of an odd function will always have symmetry with respect to the origin**. This means that a 180 degree rotation of the graph (upside-down) results in the same graph. For example, the graph of the odd function $y=x^3$ is shown below.

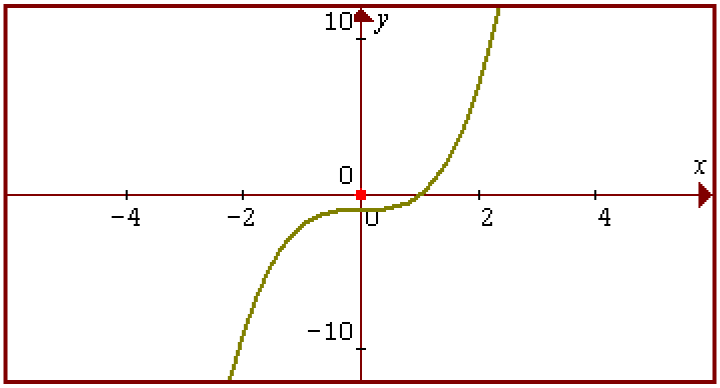


x	y
1	1
-1	-1
2	8
-2	-8
1/2	1/8
-1/2	-1/8
0	0

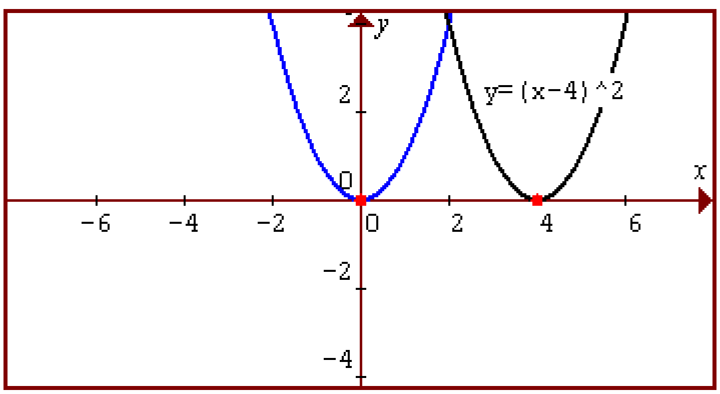
Here, opposite values of x, like 2 and –2, resulted in opposite values of y, 8 and -8. The function consisting of this single odd power of x will always result in opposite y-values when opposite x-values are input. The graph has symmetry with respect to the origin since spinning it around 180 degrees about the origin results in the same graph.

Example: Is the function $f(x) = x^3 - 1$ odd?
An easy way to show that a function is not odd is to input two opposite x-values. In this case, if we input x=2, we get f(2) = 8 – 1 = 7 and if we input x=-2, we get –8 – 1 = -9. **Since 7 and –9 are not exact opposites, this function is not odd.**

It's graph is shown below. If you spin the graph about the origin 180 degrees, you do not get the same graph.



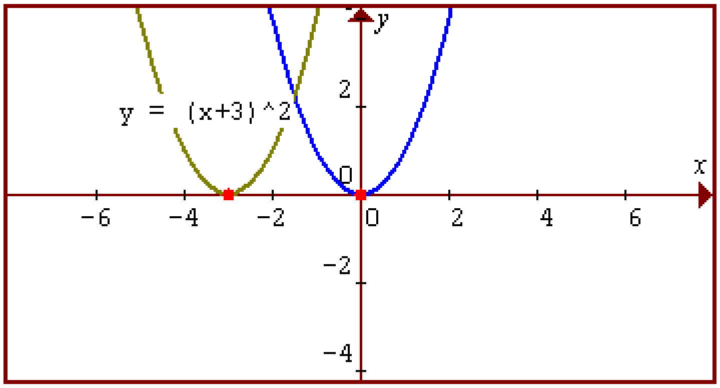
Shifting Functions Right
If f(x) is a function, we can say that g(x) = f(x-c) will have the same general shape as f(x) but will be shifted to the right "c" units. This is shown below in the comparison of the graphs of $y=x^2$ and $y = (x - 4)^2$. **The graph of $y = (x - 4)^2$ has the same shape as $y = x^2$ but is shifted 4 units to the right.**



RULE OF THUMB: If you replace each x in the formula with (x - c), your graph will be shifted to the right "c" units.

Shifting Functions Left

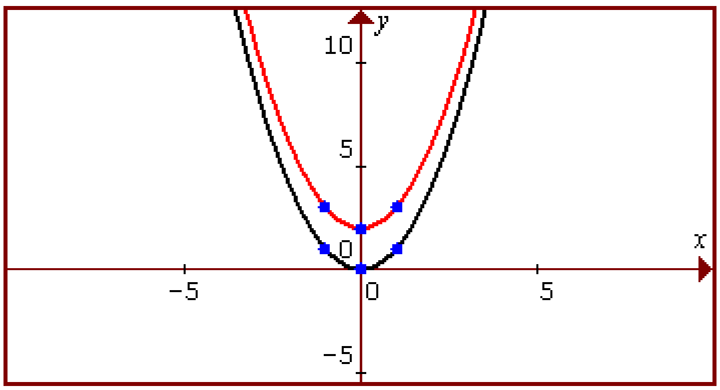
If f(x) is a function, we can say that g(x) = f(x+c) will have the same general shape as f(x) but will be shifted to the left "c" units. This is shown below in the comparison of the graphs of $y=x^2$ and $y = (x + 3)^2$. **The graph of $y = (x + 3)^2$ has the same shape as $y = x^2$ but is shifted 3 units to the left.**



RULE OF THUMB: If you replace each x in the formula with (x + c), your graph will be shifted to the left "c" units.

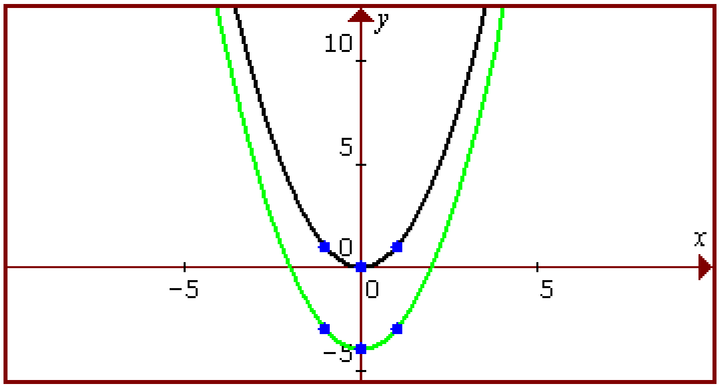
Shifting Functions Up

If f(x) is a function, we can say that g(x) = f(x) + c will have the same general shape as f(x) but will be shifted up "c" units. This is shown below in the comparison of the graphs of $y=x^2$ and $y = x^2 + 2$. **The graph of $y = x^2 + 2$ has the same shape as $y = x^2$ but is shifted 2 units up.**



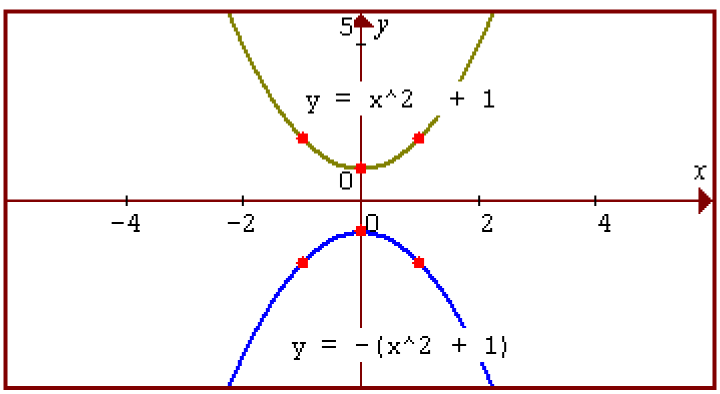
RULE OF THUMB: If you add a positive value "c" to the formula, your graph will be shifted up "c" units.

Shifting Functions Down
If f(x) is a function, we can say that g(x) = f(x) -c will have the same general shape as f(x) but will be shifted down "c" units. This is shown below in the comparison of the graphs of $y=x^2$ and $y = x^2 - 4$. **The graph of $y = x^2 - 4$ has the same shape as $y = x^2$ but is shifted 4 units down.**



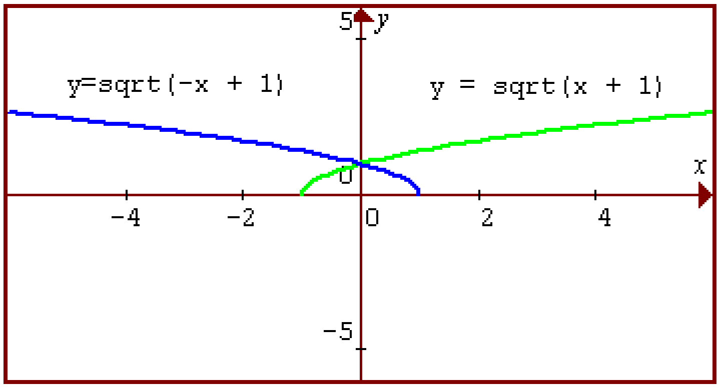
RULE OF THUMB: If you subtract a positive value "c" from the formula, your graph will be shifted down "c" units.

Reflecting Functions Across The x-axis
If f(x) is a function, we can say that g(x) = -f(x) will have the same general shape as f(x) but will be reflected across the x-axis. This is shown below in the comparison of the graphs of $y=x^2 + 1$ and $y = -(x^2 + 1)$. **The graph of $y = -(x^2 + 1)$ has the same shape as $y = x^2 + 1$ but is reflected across the x-axis.**



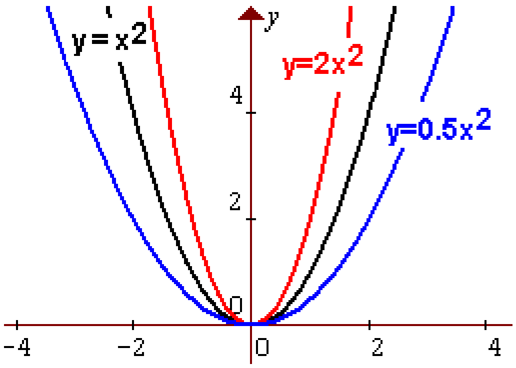
RULE OF THUMB: If you take the negative of the entire formula, your graph will be "flipped" across the x-axis.

Reflecting Functions Across The y-axis
If f(x) is a function, we can say that g(x) = f(-x) will have the same general shape as f(x) but will be reflected across the y-axis. This is shown below in the comparison of the graphs of y = √(x + 1) and y = √(-x + 1).
The graph of $y = \sqrt{-x+1}$ has the same shape as the graph of $y = \sqrt{x+1}$ except it is reflected across the y-axis.



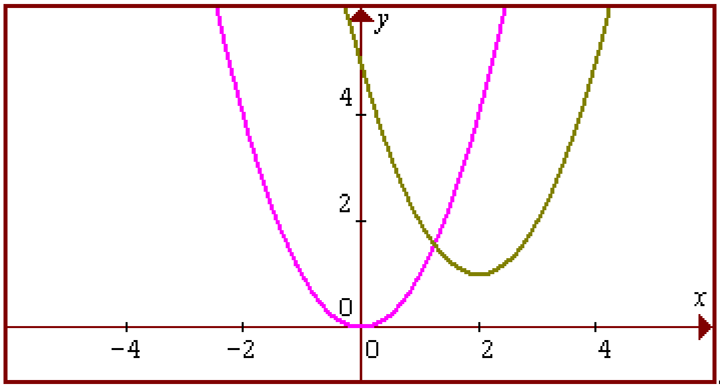
RULE OF THUMB: If you replace each x in the formula with –x, the graph will be "flipped" across the y-axis.

Vertically Stretching and Shrinking a Function
If f(x) is a function, we can say that g(x) = c•f(x) will have the same general shape as f(x) but will be vertically stretched or vertically shrunk by a factor of c. This is shown below in the comparison of the graphs of $y=x^2$, $y = 2x^2$, and $y = 0.5x^2$ **The graph of $y = 2x^2$ is vertically stretched by a factor of 2 and the graph of $y=0.5x^2$ is vertically shrunk by a factor of 0.5 .**



RULE OF THUMB: If you multiply the entire formula by a constant, your graph will be vertically stretched out or shrunk, depending on the value of the constant.

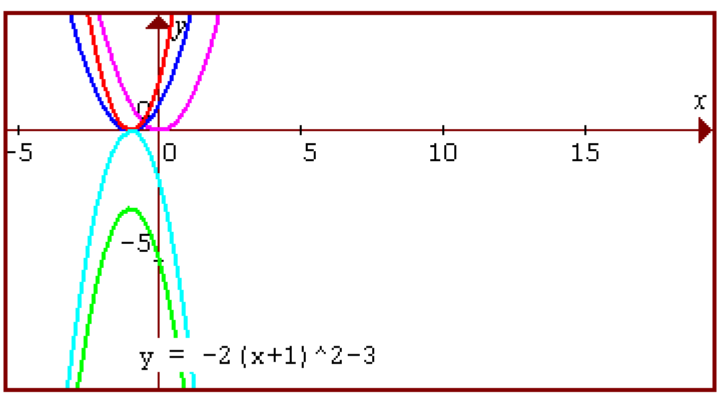
Using More Than One Function Shift Rule At a Time
Often, you will use 2 or more of the function shift rules to predict what a graph will look like and where it will be located. For example, the graph of $y = (x-2)^2 + 1$ is a shift of $y=x^2$ right 2 and up 1 as shown below



Example: Use function shift rules to predict what the graph of $y = -2(x + 1)^2 – 3$ will look like. We start with the function $y = x^2$.

$y = (x + 1)^2$ will be a shift of $y=x^2$ left 1.
y = 2(x + 1)2 will be a vertical stretch of $y = (x + 1)^2$ by a factor of 2.
$y = -2(x + 1)^2$ will be a reflection of f $y = 2(x + 1)^2$ across the x-axis.
$y = -2(x + 1)^2 –3$ will be a shift of $y = -2(x + 1)^2$ down 3 units.

So in summary, shift left, vertically stretch, reflect, and shift down. The graphs you get through all these steps are shown below:



TIP: Always do the vertical shift last. By the order of operations, the addition and subtraction operations associated with a vertical shift is done after the other operations.