Exponents and Radicals Review

Exponent Properties

$x^m x^n = x^{m+n}$

$\frac{x^M}{x^N} = x^{M-N}$

$(x^M)^N = x^{M \cdot N}$

$x^{-N} = \frac{1}{x^N}$

$\frac{1}{x^{-N}} = x^N$

$x^{\frac{M}{N}} = \sqrt[N]{x^M} = \left(\sqrt[N]{x}\right)^M$

Examples

$x^5 \cdot x^7 = x^{5+7} = x^{12}$

$\frac{X^{14}}{X} = \frac{X^{14}}{X^1} = X^{14-1} = X^{13}$

$(x^5)^7 = x^{5 \cdot 7} = x^{35}$

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

$16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

Radical Properties

$\sqrt{x^2} = x$ if $x \geq 0$

$\sqrt{x^2} = -x$ if $x < 0$

$\sqrt{x^2} = \pm x$ if x is unknown

$\sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$

$\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}}$

$\frac{1}{\sqrt{X}} = \frac{1}{\sqrt{X}} \cdot \frac{\sqrt{X}}{\sqrt{X}} = \frac{\sqrt{X}}{X}$

$\frac{1}{A+\sqrt{B}} = \frac{1}{A+\sqrt{B}} \cdot \frac{A-\sqrt{B}}{A-\sqrt{B}} = \frac{A-\sqrt{B}}{A^2 - B}$

$\frac{1}{A-\sqrt{B}} = \frac{1}{A-\sqrt{B}} \cdot \frac{A+\sqrt{B}}{A+\sqrt{B}} = \frac{A+\sqrt{B}}{A^2 - B}$

Examples

$x^2 = 5$ has solutions $x = +/- \sqrt{5}$

$\sqrt{200} = \sqrt{100} \cdot \sqrt{2} = 10 \cdot \sqrt{2} = 10\sqrt{2}$

$\sqrt{\frac{4}{49}} = \frac{\sqrt{4}}{\sqrt{49}} = \frac{2}{7}$

$\frac{x}{\sqrt{2x}} = \frac{x \cdot \sqrt{2x}}{\sqrt{2x} \cdot \sqrt{2x}} = \frac{x \cdot \sqrt{2x}}{2x} = \frac{\sqrt{2x}}{2}$

MORE EXAMPLES

Example: Rationalize the Denominator of $\dfrac{\sqrt{3}+\sqrt{2}}{1-\sqrt{5}}$

$$\frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{5}} = \frac{(\sqrt{3}+\sqrt{2})(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})} = \frac{\sqrt{3}\cdot 1+\sqrt{3}\cdot\sqrt{5}+\sqrt{2}\cdot 1+\sqrt{2}\cdot\sqrt{5}}{1\cdot 1+1\cdot\sqrt{5}-\sqrt{5}\cdot 1-\sqrt{5}\cdot\sqrt{5}}$$

$$= \frac{\sqrt{3}+\sqrt{15}+\sqrt{2}+\sqrt{10}}{1+\sqrt{5}-\sqrt{5}-5} = \frac{\sqrt{3}+\sqrt{15}+\sqrt{2}+\sqrt{10}}{-4}$$

$$= -\frac{\sqrt{3}+\sqrt{15}+\sqrt{2}+\sqrt{10}}{4}$$

Example: Simplify $\dfrac{(-2x^2y^3)^2}{(-4x^2y)^3}$

$$= \frac{(-2)^2\cdot(x^2)^2\cdot(y^3)^2}{(-4)^3\cdot(x^2)^3\cdot y^3}$$

$$= \frac{4x^4y^6}{-64x^6y^3} = -\frac{y^3}{16x^2}$$

Example: Evaluate $\left(\dfrac{8}{27}\right)^{\frac{2}{3}}$

$$= \frac{8^{\frac{2}{3}}}{27^{\frac{2}{3}}}$$

$$= \frac{\left(\sqrt[3]{8}\right)^2}{\left(\sqrt[3]{27}\right)^2} = \frac{(2)^2}{(3)^2} = \frac{4}{9}$$

Example: Simplify $\sqrt[3]{16x^2y^8}$

$$= \sqrt[3]{2 \cdot 8 \cdot x^2 \cdot y^2 \cdot y^6}$$

$$= \sqrt[3]{2 \cdot 2^3 \cdot x^2 \cdot y^2 \cdot (y^2)^3}$$

$$= \sqrt[3]{2^3} \cdot \sqrt[3]{(y^2)^3} \cdot \sqrt[3]{2 \cdot x^2 \cdot y^2}$$

$$= 2y^2 \sqrt[3]{2x^2y^2}$$

Example: Simplify $\sqrt{12x^2y^7}$

$$= \sqrt{4 \cdot 3 \cdot x^2 \cdot y^6 \cdot y}$$

$$= \sqrt{4} \cdot \sqrt{x^2} \cdot \sqrt{(y^3)^2} \cdot \sqrt{3 \cdot y}$$

$$= 2|x|y^3\sqrt{3y}$$

Note that the absolute value bars insure that the square root is a positve value.

Example: Rationalize the Denominator of $\frac{6}{\sqrt[3]{9x^2}}$

$$= \frac{6}{\sqrt[3]{9x^2}} \cdot \frac{\sqrt[3]{3x}}{\sqrt[3]{3x}}$$

$$= \frac{6\sqrt[3]{3x}}{\sqrt[3]{9x^2 \cdot 3x}}$$

$$= \frac{6\sqrt[3]{3x}}{\sqrt[3]{27x^3}}$$

$$= \frac{6\sqrt[3]{3x}}{3x}$$

$$= \frac{2\sqrt[3]{3x}}{x}$$