

Elementary Graphing, Distance Formula, & Graphs of Circles

Graphing in the plane consists of plotting coordinate pairs. In this section we will see how the distance formula is used.

Graphing Points

When graphing a point (x, y) , the first coordinate defines the location on the horizontal axis and the second coordinate defines the location on the vertical axis. So a point $(3, -5)$ would have a horizontal coordinate of 3 and a vertical coordinate of -5 .

Elementary Graphing of Equations

If you are graphing equations for the first time and you have no specialized method to use, you can graph many equations by doing the following:

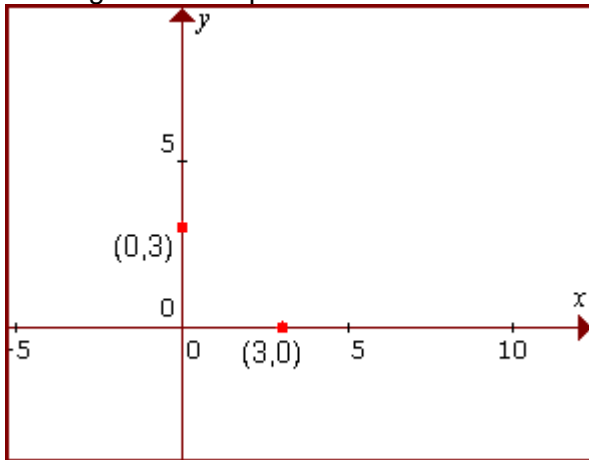
1. Find and plot all y-intercepts. To find these let $x = 0$ and solve for y . Then find and plot all x-intercepts. To find these, let $y=0$ and solve for all values of x .
2. Find and plot more points on in between the intercepts you plotted and on each side of the intercepts you plotted.
3. Draw a smooth curve through your plotted points from left to right.

Example: Graph $2y + 2x = 6$

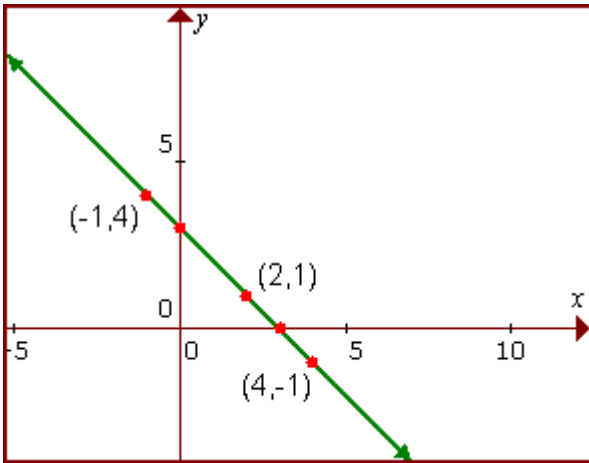
If $x=0$, $2y + 2(0) = 6$ which simplifies to $2y=6$, with $y=3$. The x-intercept is $(0,3)$.

If $y=0$, $2(0) + 2x = 6$ which simplifies to $2x=6$, with $x=3$. The y-intercept is $(3,0)$.

Plotting these two points results in



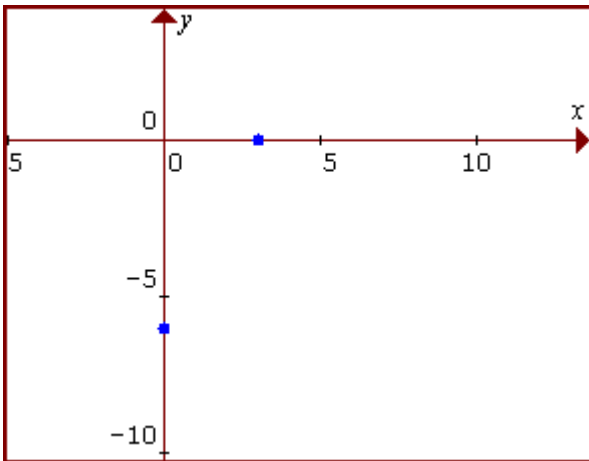
Now, find some more points. Plug in $x=2$, $x=4$, and $x=-1$ and find the corresponding y -values. For $x=2$ you get $2y + 4 = 6$ with $y=1$, for $x=4$ you get $2y + 8 = 6$ with $y=-1$, and for $x=-1$ you get $2y - 2 = 6$ with $y=4$. Pairing up your values results in the points $(2,1)$, $(4,-1)$, and $(-1, 4)$. Plot these points, and then draw a smooth curve through – in this case we end up with a straight line as shown on the next page.



Example: Graph $y = -2|x - 3|$

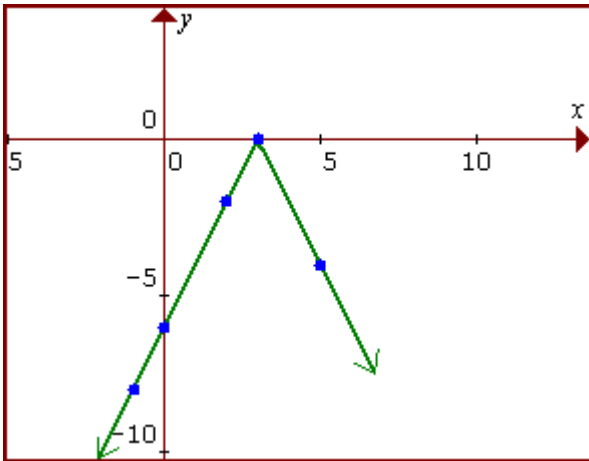
To find the y-intercept, we let $x=0$, resulting in $y = -2|0 - 3| = -2|-3| = -2(3) = -6$. Note that we took absolute value of -3 here. So the y-intercept is $(0, -6)$.

To find the x-intercept, we let $y=0$, resulting in $0 = -2|x - 3|$. Apply the Division Property of Equality to divide both sides by -2 to get $0 = |x - 3|$. The two cases for this are really the same case. We solve $0 = x - 3$ to get $x = 3$. So the x-intercept is $(3, 0)$. The points are plotted and shown below.



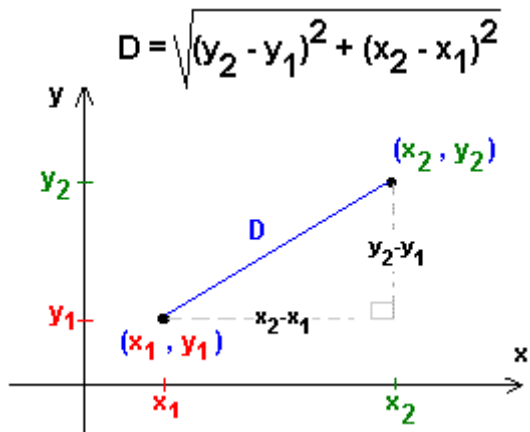
Now, you need to find some more points to see what this graph is doing. Some good values of x to use to find more points would be $x=2$, $x=5$, and $x=-1$ since these will give us points on each side of the intercepts.

Plugging $x=2$, $x=5$, and $x=-1$ into $y = -2|x - 3|$ results in $y=-2$, $y=-4$, and $y=-8$, respectively, so the points are $(2, -2)$, $(5, -4)$, and $(-1, -8)$. Plot these points and draw a curve through from left to right, as shown on the next page.



The Distance Formula

The distance formula states:



where D is the distance between (x_1, y_1) and (x_2, y_2) . As may be seen in the graph above, the distance from point to point really is the hypotenuse of a right triangle of sides length $(x_2 - x_1)$ and $(y_2 - y_1)$.

To use the Distance Formula, simply plug the values of (x_1, y_1) and (x_2, y_2) into the formula.

Example: Find the distance from $(4, -1)$ and $(-2, 2)$.

Plug the values from $(\overset{x_1}{4}, \overset{y_1}{-1})$ and $(\overset{x_2}{-2}, \overset{y_2}{2})$ into

$$D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

To get

$$D = \sqrt{(2 - (-1))^2 + (-2 - 4)^2}$$

$$D = \sqrt{45}$$

or $3\sqrt{5}$ if you simplify the radical.

Equation of a Circle

A circle of radius r , centered at (h,k) is defined as all points (x, y) that are “ r ” units from the center (h,k) . If we apply the distance formula to this relationship, where the distance from (h,k) to (x, y) is r , we get the formula

$$(x - h)^2 + (y - k)^2 = r^2$$

Example: If a circle has a center at $(3,-1)$ with radius 2, what is its equation?

Here, $h=3$, $k = -1$, and $r = 2$. We substitute these values into the circle formula to get

$$(x - 3)^2 + (y - (-1))^2 = 2^2 \text{ which simplifies to}$$

$$(x - 3)^2 + (y + 1)^2 = 4$$

And here is the graph of this circle:

