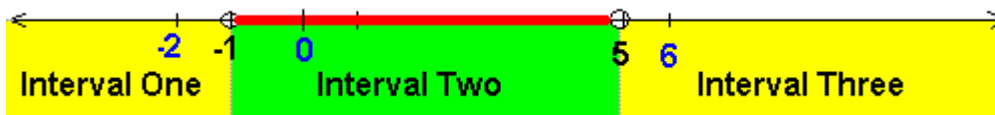


## Solving Absolute Value Inequalities

Absolute Value inequalities may be solved using the general methods for solving inequalities – see <http://www.mathmotivation.com/lectures/Inequalities.pdf>. In summary you replace the inequality symbol with =, solve this equation to find the critical numbers, plot the critical numbers, and test the intervals. For example, the inequality  $|x - 2| < 3$  may be solved by first solving  $|x - 2| = 3$  to get  $x=5$  and  $x=-1$ . (See <http://www.mathmotivation.com/lectures/Absolute-Value-Equations.pdf>). Then, plot the critical numbers  $x = 5$  and  $x = -1$  on the number line and check the intervals.



The test value of Interval Two,  $x=0$  results in a true statement when substituted into  $|x - 2| < 3$  whereas the test values of Interval One and Interval Three,  $x = -2$  and  $x = 6$ , result in false statements. So Interval Two, makes up the solution, i.e.  $-1 < x < 5$ .

But, there is a shortcut. And provided you use this shortcut for correctly, you will get the correct solutions with a bit less work.

### A Shortcut For Less-Than Absolute Value Inequalities

When solving absolute value inequalities of the form

$|\text{Algebraic Expression}| < k$ , where  $k$  is some positive real number, simply rewrite this as  $-k < \text{Algebraic Expression} < k$ . Then solve.

NOTE: This same procedure applies to  $\leq$  absolute value inequalities

**Example: Solve  $|x - 2| < 3$**

$|x - 2| < 3$  may be rewritten as  $-3 < x - 2 < 3$ . Now solve this  
 $-3 + 2 < x - 2 + 2 < 3 + 2$                       Addition Property of Inequalities  
 $-1 < x < 5$     Combine Like Terms

Pretty easy huh!

### Why Does This Shortcut Work?

The explanation for this “shortcut” is that when the absolute value of a quantity is less than a positive real number, the quantity itself must be “sandwiched in between” the positive and negative values of the real number on the number line. For example, if  $|x| < 3$ , then  $x$  must be between  $-3$  and  $3$  if plotted on the number line – this is because *absolute value is defined as the distance from zero on the number line*. So  $x$  must be within 3 units of zero on the number line.

Note that this method **ONLY** applies to  $<$  inequalities! A different method is used for  $>$  inequalities.

**The Shortcut for Greater-Than Absolute Value Inequalities Is On Next Page...**

## A Shortcut For Greater-Than Absolute Value Inequalities

When solving absolute value inequalities of the form

$|\text{Algebraic Expression}| > k$ , where  $k$  is some positive real number, rewrite this as

$\text{Algebraic Expression} > k$  **AND ALSO**

$\text{Algebraic Expression} < -k$

Then solve **BOTH** inequalities. The solution will consist of the Union of two solutions.

Note: This same procedure applies to  $\geq$  absolute value inequalities.

**Example: Solve  $|2x + 1| \geq 4$**

Rewrite as  $2x + 1 \geq 4$  **AND**  $2x + 1 \leq -4$ .

Now, solve both of these inequalities.

$2x + 1 \geq 4$       **Given**

$2x \geq 3$                       **Add  $-3$  to both sides using Addition Property of Inequalities**

$x \geq 3/2$                       **Divide both sides by 2 using Division Property of Inequalities**

$2x + 1 \leq -4$       **Given**

$2x \leq -5$                       **Add  $-1$  to both sides using Addition Property of Inequalities**

$x \leq -5/2$                       **Divide both sides by 2 using Division Property of Inequalities**

The solution is the Union of these two solutions, which we write as

$x \geq 3/2$  **OR**  $x \leq -5/2$ .

Note that the term Union implies we use the word "OR" between the two solutions.

### Why Does This Shortcut Work?

The explanation for this "shortcut" is that when the absolute value of a quantity is greater than a positive real number, the quantity itself must be to the right of the positive value of the real number and left of the negative value of the real number on the number line.

For example, if  $|x| > 3$ , then  $x$  must be to the right of 3 **OR** to the left of  $-3$  when plotted on the number line – this is because *absolute value is defined as the distance from zero on the number line*. So  $x$  must be more than 3 units away from zero on the number line in either the positive or negative direction.

### In Summary:

Use these shortcuts, but use them correctly! If you are the type that would rather have fewer methods to remember, then you can use the interval method that applies to all inequalities.